

Study sheet for Final CS1100 by Matt in Wed night class

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Sequences – have patterns;

possible patterns are:

- 1. Term related to other terms**
- 2. Term described to position in sequence (eg: 1st, 2nd, 3rd)**
- 3. Multiples of 2, 10, etc.**

Notation:

$a_1 = 1^{\text{st}}$ term

$a_2 = 2^{\text{nd}}$ term

$a_3 = 3^{\text{rd}}$ term

**Recursive formula – term defined in relation to the previous term
(write out the above 25 times)**

$$a_5 = a_{5-1} + 2 \text{ or in notation } a_n = a_{n-1} + 2$$

**Closed formula – term defined by relation to its position
(write out the above 25 times)**

$$a_n = n * 2; a_3 = 3 * 2; a_5 = 5 * 2$$

To help discover a sequence, build a table:

n=	1	2	3	4	(position in the sequence)
a=	1	3	7	15	(sequence itself)

delta = **2** **4** **8** **(differences between numbers in sequence)**

Focus on the delta for clues! How is it growing?

Recursive: $2a_{n-1} + 1$ Closed: $2^n - 1$

Some common formulas: $2n, 3n, 4n, 2n+1, 3n+1, 4n+1, 2n+2, 3n+2, 4n+2, 2^n+1, 3^n+1, 4^n+1, 2n-1, 3n-1, 4n-1, 2n-2, 3n-2, 4n-2$

$n!$ -> n factorial - $3! = 3 * 2 * 1$

$$0! = 1$$

Why?

$$1! = 1 * 0!$$

Divide both sides by 1:

$$\frac{1!}{1} = \frac{1 * 0!}{1}$$

$$\frac{1}{1} = \frac{0!}{1}$$

therefore:

$$1 = 0!$$

Summation – sum of the terms in a sequence (write the above 25 times)

$$1+2+3+4+5+6=21$$

Notation:

Summation
symbol tells
you that you
are to sum
terms of
sequence

$$\sum_{i=1}^n$$

Ending
position in
sequence

Closed
formula for
sequence

Starting
position in
sequence

Example:

$$\sum_{i=1}^5 i+2$$

$$3+4+5+6+7= 25$$

Logic

A statement can be either True or False

3 major operations:

- And (\wedge): $p \wedge q$
- Or (\vee): $p \vee q$
- Implication (\rightarrow): $p \rightarrow q$

AND table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Hint: both must be true in order for statement to be True; all others result in False

OR Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Hint: ONLY one must be true in order for statement to be True

Implication table

p	q	p->q
T	T	T
T	F	F
F	T	T
F	F	T

The Rule is: if the phone rings, we MUST pick it up. If the phone rings, and we do NOT pick it up, the whole statement is False. All others are true (because we do not violate the rule). Write out 10 times.

Connective Not (essentially reverses the statement to the opposite)

p	¬p
T	F
F	T

Tautology - always leads to a true result

Contradiction - always leads to a false result

When faced with a longer logic statement such as:

$p \rightarrow (p \vee \neg q)$

break it into parts in a table: p, q, ¬q, $p \vee \neg q$, $p \rightarrow (p \vee \neg q)$

Do parentheses first

To build a 3 element table:
Insert a standard T & F table 2 times:

p	q	r
T	T	T
T	T	F
F	T	T
F	T	F
T	F	T
T	F	F
F	F	T
F	F	F

Then add all True for the 1st four lines, then all False for the last 4:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Sets

A set is a group of objects with no duplicates, separated by commas:

$\{1,2,3,4\}$

A set can contain other sets:

$\{1,2,\{\text{cameron, bob}\}, 3, 4\}$

Sets can be empty:

$\{ \}$

Order does not matter:

$\{4,3,6,1,9\}$

Can name a set:

$A = \{1,2,4,16,256\}$

Cardinality - refers to the size of the set:

$A = \{x,y,z\}$

$|A| = 3$ (cardinality of A is 3)

$E = \{\{a,b\}, \{c,d\}\}$

$|E| = 2$ (each set is an element; 2 elements)

Infinite Sets

“N” - natural numbers: $\{0,1,2,3\dots\infty\}$

“Z” - integers: $\{-2,-1,0,1,2\}$

“Q” - rational numbers: numbers that can be expressed as a fraction: $1/3, 1/4, 1/5$

“R” - real numbers; all numbers, fractions, decimals

Sets (continued)

$$A = \{x/2: x=y*2 \wedge y=\{-2,-1,0,1\}\}$$


Read from right to left: put numbers into the y formula, then put results (“x”) into x formula.

Cartesian product:

$$A \times B$$

Take each element in 1st set and combine it with each element in 2nd set, eg: $\{1,2\} \times \{4,5\} = \{(1,4), (1,5), (2,4), (2,5)\}$

Operations with Sets

Union: $A \cup B$ means to join 2 sets together; no duplicates

$$A = \{c,d\} \quad B = \{x,y\} \quad A \cup B = \{c,d,x,y\}$$

Intersection: $A \cap B$ - where 2 sets meet or have common values

$$A = \{c,d,e,f\} \quad B = \{e,f,g,h\}$$

$$A \cap B = \{e,f\}$$

Notation:

$$\{x : x \in N\}$$



: means subset of a larger set



: means element of a set

$x: x=2k \ \& \ k \in \{1,2,3\} = \{2,4,6\}$ - x is made up of $2k$ and the elements of set k is $\{1,2,3\}$; therefore, $2*1=2$, $2*2=4$, $2*3=6$

Difference between sets:

$A - B$: one takes the elements of B and removes them from A .

$$A = \{1,2,3,4\} \quad B = \{2,4\} \quad A - B = \{1,3\}$$

Power Sets

Power sets are sets that have taken steroids. 😊

Power sets include all the subsets of the main set, including the empty set:

$$A = \{1, 2, 3\}$$

$$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Another way to indicate a Power set is: 2^A

Cartesian Product

A set that contains all pairs of coordinates:

$$A = \{1, 2\} \quad B = \{y, z\}; \quad A \times B = \{(1, y), (1, z), (2, y), (2, z)\}$$

Set Difference

$A - B$ = takes elements of Set B and removes them from Set A

Functions

In order to be a function, all results of the rule must be in the codomain

Function Properties:

- 1. onto: each value in the codomain can be produced by a value in the domain**
- 2. one-to-one: one and only one value in the Domain points to one value in the Codomain**

Inverses

The inverse of a function is another function that reverses the process of the original function.

To create an inverse:

- Make the old Codomain the new domain**
- Make the old domain the new Codomain**
- Swap the $f(x)$ and the x in the formula**
- Use algebra to get the $f(x)$ back by itself**


Function Composition (o)

Chains together two functions.

$f \circ g$ which reads “f compose g”.

The result of the second function is plugged into the first function.

Another way of writing it is: $f(g(x))$; start on the inside and work outwards:

$$f(g(x)) = f(x) \underbrace{g(x)}$$
A diagram illustrating the substitution in the function composition equation. A curly brace is drawn under the expression $g(x)$ in the equation $f(g(x)) = f(x) \underbrace{g(x)}$. An arrow starts from the right side of this brace and points to the x in the $f(x)$ term on the left side of the equation.

Take result of $g(x)$ and put it into the “x” in $f(x)$

Probability

Set of Outcomes

This is the set of outcomes that we are interested in occurring.

Sample Space

This is the set of every possible outcome.

Set of Outcomes

Sample Space

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		Order?	
		Yes	No
Duplicates?	Yes	<u>Ordered List</u> n^r n = size of the list r = # ways to select elements	<u>Unordered List</u> $\frac{P(n+r-1, r)}{r!}$
	No	<u>Permutation</u> $P(n, r) = \frac{n!}{(n-r)!}$	<u>Set</u> $C(n, r) = \frac{P(n, r)}{r!}$

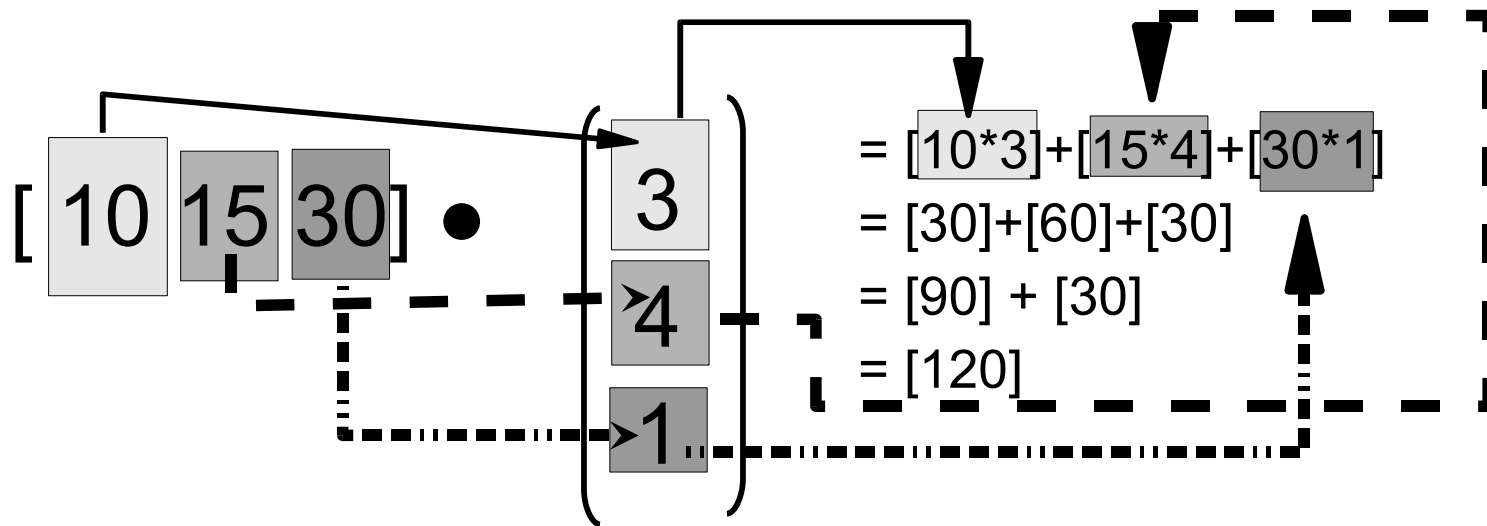
Write each formula 25 times. Start with the Ordered List (n^r) then do Permutation, Set, Unordered List, since they build upon each other.

Then put it together by writing out the above chart, starting with Ordered List, then Permutation, Set and finally Unordered List (follow dotted lines.). On the final, write this chart down as soon as the test is started.

Multiplying Single Dimension Arrays

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Multiplying Single Dimension Matrices

$$\begin{aligned} [2 \ 4 \ 16 \ 32 \ 64] \bullet \begin{pmatrix} 64 \\ 32 \\ 16 \\ 4 \\ 2 \end{pmatrix} &= [2 \cdot 64] + [4 \cdot 32] + [16 \cdot 16] + [32 \cdot 4] + [64 \cdot 2] \\ &= [128] + [128] + [256] + [128] + [128] \\ &= [256] + [256] + [256] \\ &= [512] + [256] \\ &= [768] \end{aligned}$$

$$[-2 \ -4 \ 16 \ 32] \bullet \begin{pmatrix} -32 \\ -16 \\ -4 \\ 2 \end{pmatrix}$$

$$= [-2 \cdot -32] + [-4 \cdot -16] + [16 \cdot -4] + [32 \cdot 2]$$

$$= [64] + [64] + [-64] + [64]$$

$$= [128]$$

$$[-3 \ -1 \ -2] \bullet \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$= [-3 \cdot 2] + [-1 \cdot 1] + [-2 \cdot 3]$$

$$= [-6] + [-1] + [-6]$$

$$= [-13]$$

$$[-1 \ 3 \ -2 \ 5] \bullet \begin{pmatrix} -6 \\ 9 \\ -2 \\ 4 \end{pmatrix}$$

$$= [-1 \cdot -6] + [3 \cdot 9] + [-2 \cdot -2] + [5 \cdot 4]$$

$$= [6] + [27] + [4] + [20]$$

$$= [37] + [20]$$

$$= [57]$$